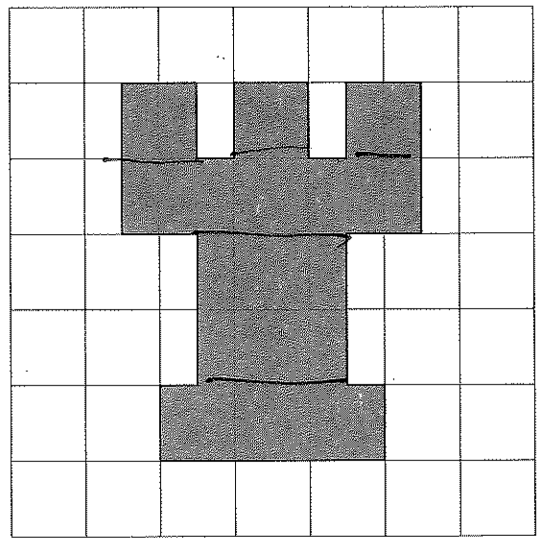
Name: Answer Key Section 002

MA 202 EXAM 2: March 6, 2018

Instructions: The following exam has 90 possible points. The point value of each question is stated explicitly. No books or notes may be used on this exam. Please **write legibly** and keep your paper as organized as possible. You **may not** use a calculator on this exam. **Show all your work!** Answers without explanation will not receive full credit. Use complete sentences where appropriate. If you have any questions, be sure to ask. Good luck!

Question	Points	Score
1	10	
2	8	
3	6	
4	6	
5	12	
6	12	
7	16	
8	12	
9	8	
Bonus	4	
Total:	90	

1. (10 points) On the grid below, each square is 1 unit × 1 unit.



(a) Find the perimeter of the figure. Give an exact answer.

Counting units, we get $\boxed{24 \text{ units}}$ +4

(b) Find the area of the figure. Give an exact answer.

Break into smaller rectangles or count squares:

$3(1\times 1) + 4\times 1 + 2\times 2 + 3\times 1 = 3+4+4+3 = \boxed{14 \text{ units}^2}$ +4

(c) Suppose the figure is used to make a 3-dimensional solid by stacking three 1 × 1 × 1 cubes on each square and three half cubes on each half square (i.e. make a solid whose base is the figure shown above and is 3 units tall). Determine the volume of this solid by counting cubes.

Each of the 3 layers contains 14 unit cubes, so $V = 3\cdot 14 = \boxed{42 \text{ units}^3}$ +2

2. (8 points) You are building a snow globe model of the Parisian skyline with a radius of 30mm.

(a) Determine the volume of the *upper hemisphere* of the snow globe. Give an exact answer.

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(30)^3 = \frac{2}{3}\pi(\cancel{27}^{9},000) = \boxed{18{,}000\,\pi\ mm^3}$$

+1 +1 +1 +1

(b) If 75% of the upper hemisphere is filled with liquid, determine, in **cubic centimeters**, how much of the upper hemisphere is occupied by the solid portion of the model. (Hint: If there are are 10 mm in 1 cm, how many 1 mm × 1 mm × 1 mm cubes fit in a 1cm × 1cm × 1cm cube?)

+1 +1

25 % of $18{,}000\pi\ mm^3$ is solid : $\frac{18000\pi}{4} = 4500\,\pi\ mm^3$

+1

There are $10 \cdot 10 \cdot 10 = 1000\ mm^3$ in $1cm^3$, so the solid portion occupies

$$4500\,\pi\ mm^3 \cdot \frac{1\ cm^3}{1000\ mm^3} = \boxed{4.5\,\pi\ cm^3}$$

+1

3. (6 points) Determine if the following statements are *always*, *sometimes*, or *never* true. Explain your reasoning.

(a) Doubling the width of a rectangle doubles the perimeter of the rectangle.

Never. The length is not doubled and $P_{rect} = 2l + 2w$

+2 +2

(b) For a cylinder and cone with congruent bases and of the same height, the ratio of the volume of the cylinder to the volume of the cone is $\frac{1}{3} : 1$.

Never. Since the bases are congruent, their area B is the same. So

+2

$V_{cyl} : V_{cone}$

$Bh : \frac{1}{3} Bh$ +2

$1 : \frac{1}{3} \neq \frac{1}{3} : 1$.

4. (6 points) You decide to wrap a 3D puzzle in the shape of a cube to mail to your mom for her birthday. You have the following two packaging options.

- **Option 1:** Wrap the puzzle as a whole and mail it in one piece.
- **Option 2:** Break the puzzle in half into two smaller rectangular prisms, wrap each separately, and mail the two individual pieces.

(a) Which option uses the least amount of wrapping paper? *Explain* your reasoning.

+1 Option 1. Breaking the cube into smaller prisms preserves lateral surface area but exposes two new faces, requiring more paper to wrap.

+2

(b) The local postal service charges by volume. Mailing packages costs \$0.57 per cubic inch. Which option is cheaper to mail? *Explain* your reasoning. +3

They cost the same, since breaking the puzzle in half does not change its volume.

5. (12 points) The area of a rectangle with base b and height h is $A = bh$. In this problem you will use this information to derive the area formulas for two different polygons. For each, your solution should include complete sentences and a labeled picture.

(a) Circle *one* of the following two polygons for your first derivation:

Parallelogram Triangle

i. Area formula: ____________________

ii. Labeled Picture:

See section 3 answer key

iii. Explanation/Derivation of Formula:

(b) Circle *one* of the following polygons for your second derivation. Here, you may also use the area formula derived in the previous part.

Trapezoid Kite

i. Area formula: ____________________

ii. Labeled Picture:

iii. Explanation/Derivation of Formula:

You may use any of the following conversions to answer the following question.

- 1 minute = 60 seconds
- 1 hour = 60 minutes
- 1 mile ≈ 1.61 kilometers
- 1 kilometer ≈ 0.62 miles
- 1 kilometer = 1000 meters
- 1 pound = 16 ounces
- 1 cup = 8 fluid ounces
- 1 gallon = 16 cups
- 1 gallon ≈ 4 liters
- 1 liter ≈ 0.25 gallons

6. (12 points) The current world record holder in the men's mile is Hicham El Guerrouj of Morocco with a time of 3:43.13 while the world record holder in the women's mile is Svetlana Masterkova of Russia with a time of 4:12.56.

(a) Will Svetlana run the 1600m race faster or slower than her record in the mile? Will Hicham run the 1600m race faster or slower than his record in the mile? Explain.

Both will be slightly faster, since 1600m = 1.6 km is slightly shorter than 1.61 km ≈ 1mi

+1 +1 +1 conversion

(b) Round Svetlana's time to 4 minutes and 12 seconds. If it were possible to sustain this pace, how many minutes would it take Svetlana Masterkova to run a half marathon (i.e. 13.1 miles)?

At 1 mile in 4 min 12 s = $4 + \frac{12}{60} = 4.2$ min, it would take $13.1 \cdot 4.2 = \boxed{55.02 \text{ min}}$.

+1 +1 +1

$$\begin{array}{r} 13.1 \\ \times\ 4.2 \\ \hline 262 \\ 5240 \\ \hline 55.02 \end{array}$$

(c) Round Hicham's time to 3 minutes and 45 seconds. If it were possible to sustain this pace, how many miles would Hicham El Guerrouj run in one hour?

Hicham runs at a speed of $1\text{mi}/3\text{min}\,45\text{s} = 1\text{mi}/3\tfrac{45}{60}\text{min} = \frac{1\text{mi}}{\frac{15}{4}\text{min}} = \frac{4\text{ mi}}{15\text{ min}}$.

+1

So in 1 hr = 60 min he will run $60 \cdot \frac{4}{15} = \boxed{16 \text{ mi}}$

+1 +1

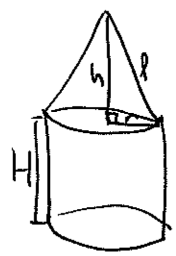
(d) During his weekly 20 mile long-run, Hicham drank 3 liters of Gatorade. If one bottle of Gatorade contains 32 fluid ounces, how many bottles did Hicham drink?

$3\,L \approx \frac{3}{4}\,gal = 12\,c = 96\,oz =$ $\boxed{\text{3 bottles of Gatorade}}$

+1 +1 +1

7. (16 points) Rapunzel is trapped in a tower that is comprised of a right circular cylinder of height H and radius r and a roof that is the shape of a cone of radius r and height h.

(a) Determine the slant height, l, of the roof by using the right triangle with hypotenuse l, base r, and height h. (It might be helpful to sketch the composite figure that forms the tower).



By Pyth. Thm:

$h^2+r^2=l^2$, so $l=\sqrt{h^2+r^2}$

+2 +2

(b) If $h = 8$ meters and $r = 6$ meters, approximate (using $\pi \approx 3.14$) the lateral surface area of the cone roof.

$l=\sqrt{8^2+6^2}=\sqrt{100}=10m$ +1

$A=\pi r l = \pi\cdot 6\cdot 10 = 60\pi\ m^2$

+2

$\approx \boxed{188.4\ m^2}$

+1

$$\begin{array}{r} 3.14 \\ \times\ 60 \\ \hline 000 \\ 18840 \\ \hline 188.40 \end{array}$$

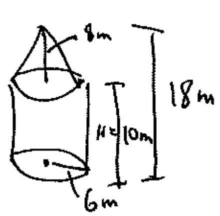
(c) Last week a crumple-horned snorkack attack destroyed the roof of her tower and you must now re-shingle it. If shingles cost approximately $5.50 per square meter, determine how much it will cost to repair her roof. Round to the nearest hundredth.

Cost $= 188.4\ m^2 \cdot \$5.50/m^2$ +3

$= \$1036.20$ +1

$$\begin{array}{r} 188.4 \\ \times\ 5.5 \\ \hline 9420 \\ 94200 \\ \hline 1036.20 \end{array}$$

(d) If the tower (from the ground to the highest point of the roof) is 18 meters tall, find the volume of the tower. (Again it may be helpful to have a sketch of the tower.)



so $8+H=18m$, i.e. $H=10m$. +1

$V=V_{cone}+V_{cyl}$ +1

$=\frac{1}{3}\pi r^2 h + \pi r^2 H$

$=\frac{1}{3}\pi(6)^2(8)+\pi(6)^2 10$

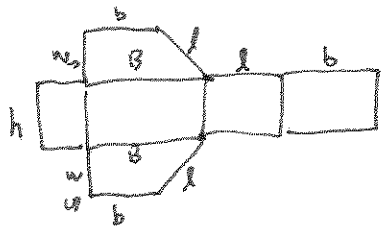
$=96\pi + 360\pi$

$=\boxed{456\pi\ m^3}$

8. (12 points) Consider a right trapezoidal prism with height h and a base with parallel sides b and B and width w.

2 (a) Sketch a net of the prism described above, and label all the given information.

~~2 points~~



1 pt net
1 pt labels

6 (b) If $h = 10$cm, $w = 3$cm, $b = 5$cm, and $B = 9$cm find the **surface area** *and* the **volume** of the prism. The remaining side l of the trapezoid is 5 cm long.

3 pts: $SA = 2\cdot\frac{1}{2}(b+B)w + wh + bh + Bh + lh$
$= 2\cdot\frac{1}{2}(5+9)(3) + 10(3+5+9+5)$
$= 42 + 10(22)$
$= 262\ cm^2$

$V = (\text{Base})\cdot h$
$= \frac{1}{2}(b+B)w\cdot h$
$= \frac{1}{2}(5+9)\cdot 3\cdot 10$
$= 42\cdot 5$
$= 210\ cm^3$

with wh backwards
$SA = 2\cdot\frac{1}{2}(b+B)h + hw + bw + Bw + lw$
$= (5+9)10 + (10+5+9+5)3$
$= 140 + 29\cdot 3 = 227\,cm^2$

2 (c) What must the new height H be if you wish to keep the same base, but double the volume?

Double original height to $H = 2h = 20$ cm. b/c $V = (\text{base})h$.
$\Rightarrow 2V = (\text{base})(2h)$.

1 pt

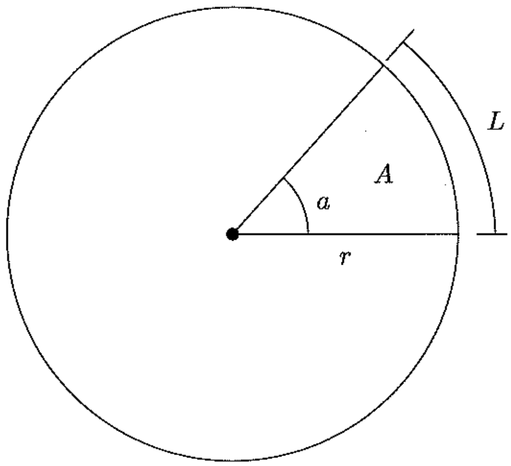
1 pt

2 (d) Will the height H from part (c) above result in the surface area doubling as well? If so, why? If not, should the height be increased by more or less to double the surface while keeping the same base?

No. The area of the rect. faces doubles but the area of the bases is unchanged, so we should increase the height by more.

1 pt.

9. (8 points) Consider the following sector of area A, arc length L, and central angle a.



(a) Set up a proportion that relates a and A.

$$\frac{a}{360^\circ} = \frac{A}{\pi r^2} \qquad +2$$

(b) If $A = 6\pi$ square inches and $a = 60$ degrees, then solve for r, the radius, in inches.

$$\frac{60}{360} = \frac{6\pi}{\pi r^2} \qquad +1$$

$$\frac{1}{6} = \frac{6}{r^2}$$

$$r^2 = 36 \qquad +1$$

$$\boxed{r = 6 \text{ in}} \qquad +1$$

(c) Use the information from part (b) to find the arc length L. Give an exact answer.

$$L = \frac{1}{6}(2\pi r) = \frac{1}{6}\cdot 2\pi \cdot 6 = 2\pi \text{ in}$$

+2 +1

10. (4 points) **BONUS:** The formula for the surface area of a prism with a square base of side length s and height h is:

$$SA = 2 \cdot s^2 + 4 \cdot sh.$$

+3 (a) Explain why the formula makes sense. Your explanation should include where the s^2 +1 and sh +1 come from and what the 4 and +1 2 count.

s^2 is the area of each square base and there are two of those. sh is the area of each lateral rectangular face and there are four of these.

+1 (b) Fill in the blank: If you quadruple the side length of the square and the height, then the surface area is increased by a factor of 16